A Mixed $\mathcal{H}_\infty$–Passivity Approach for Leveraging District Heating Systems as Frequency Ancillary Service in Electric Power Systems

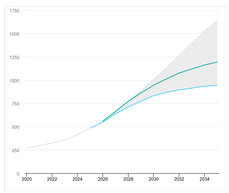
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Takeaway

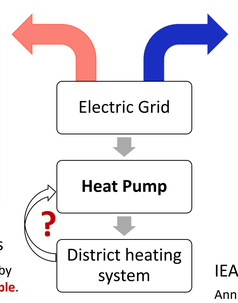
Bidirectional DHS-port shaping enables improved frequency-support dynamics while preserving EPS–DHS stability and DHS optimality.

Motivation: AI Load Growth and Thermal Flexibility

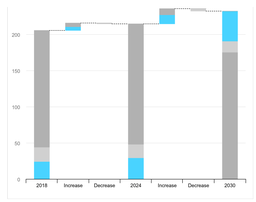


IEA (2026), Electricity consumption by data centers

Data-center demand is set to **double** by 2030 and AI-focused power use to **triple**.



Heat accounted for almost half of total final energy consumption and 37% of energy-related CO2 emissions in 2024



IEA (2025), Annual global heat use

Annual heat consumption expanded by **6%** globally.

Heat represents a major share of global energy demand, while AI-driven electricity growth increases the need for reliable grid-support resources. Heat pumps connect these two challenges and allow DHS thermal inertia to support power-system frequency.

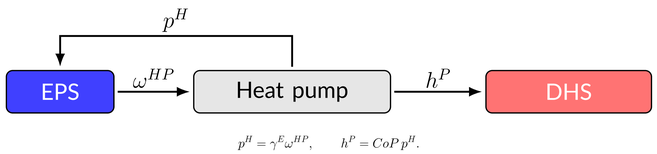
Coupled EPS–DHS Design Objectives

EPS regulation
$\omega(t) \to 0$
Restore nominal grid frequency following disturbances.

DHS optimality
$e(t) \to 0$
Recover optimal thermal operation.

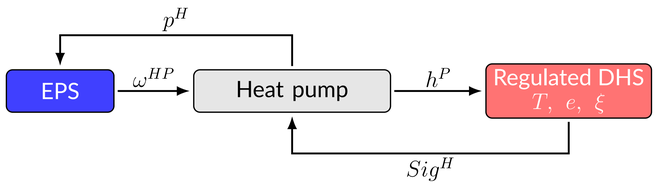
Stable coupling
$x(t) \to x^\star$
Ensure stable electro–thermal interaction.

From Fixed HP Response to a Shapeable DHS Port



$$p^H = \gamma^E \omega^{HP}, \qquad h^P = CoP\, p^H.$$

HP power is fixed by the frequency-response coefficient; DHS states do not shape the grid-facing response.



$$p^H = \gamma^E \omega^{HP} + \gamma^H Sig^H, \qquad h^P = CoP\, p^H.$$

Key idea: DHS optimality feedback makes the grid-facing port $\omega^{HP} \mapsto p^H$ shapeable.

Contributions

- Formulate the HP-connected DHS as a bidirectional electro–thermal port.
- Develop a mixed $\mathcal{H}_\infty$–passivity synthesis that ensures stable EPS–DHS interconnection and DHS optimality.
- Demonstrate improved frequency-support dynamics while preserving thermal regulation.

Key insight: Bidirectional DHS-port shaping improves frequency-support dynamics without sacrificing DHS optimality.

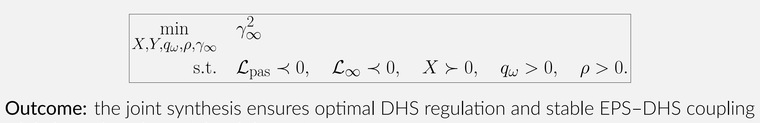
Grid-Facing EPS–DHS Port Interconnection

Augmented regulated DHS port with optimality error e:

$$\dot{T} = -AT + B_1 h^G + B_2 h^P + w^h, \qquad \dot{\xi} = e, \qquad x = \mathrm{col}(T, \xi).$$

With the disturbance-independent regulator $h^G = -Kx$,

$$\dot{x} = A_{\mathrm{cl}}(K)x + B_{\mathrm{cl}}^{(\omega)}\omega^{HP} + B_{\mathrm{cl}}^{(h)} w^h, \qquad p^H = \gamma^E \omega^{HP} + C_y(K)x.$$



EPS port: $u_1 = -p^H,\ y_1 = \omega^{HP}$.
Under the generation assumptions, the EPS with AGC is strictly passive.

DHS port: $u_2 = \omega^{HP},\ y_2 = p^H$.
The regulator K shapes the dynamic response seen by the EPS.

The grid-facing DHS port transfer matrix is

$$G_K(s) = \underbrace{C_y(K)\big(sI - A_{\mathrm{cl}}(K)\big)^{-1} B_{\mathrm{cl}}^{(\omega)}}_{G_{d,K}(s)} + \gamma^E.$$

Shapeable dynamics: $G_{d,K}(s)$ Fixed feedthrough: γ^E.

Stability and Optimality Guarantees

Optimality guarantee — Lemma 4
If A_{cl} is Hurwitz, then $e(t) \longrightarrow 0$, recovering optimal heat sharing and temperature operation.

Interconnection stability guarantee — Theorem 1
If the regulated DHS is strictly passive from ω^{HP} to p^H, then its negative-feedback interconnection with the passive EPS port is asymptotically stable.

Mixed $\mathcal{H}_\infty$–Passivity Synthesis

Introduce the convexifying variables $X = P^{-1} \succ 0, \quad Y = KX$.

Passivity condition $\mathcal{L}_{\mathrm{pas}}$: ensures that A_{cl} is Hurwitz and the regulated DHS port is strictly passive.

Weighted dynamic-port condition $\mathcal{L}_\infty$: with $W_{\mathrm{HP}}(s) = \alpha + \frac{s/\omega_h}{1+s/\omega_h},\ \omega_h > 0$, enforces $\left\| W_{\mathrm{HP}}(s) G_{d,K}(s) \right\|_\infty < \gamma_\infty$, thereby attenuating undesirable mid-/high-frequency dynamics.

Joint controller synthesis: $K = YX^{-1}$

$$\begin{aligned} \min_{X,Y,q_\omega,\rho,\gamma_\infty} \quad & \gamma_\infty^2 \\ \text{s.t.} \quad & \mathcal{L}_{\mathrm{pas}} \prec 0, \quad \mathcal{L}_\infty \prec 0, \quad X \succ 0, \quad q_\omega > 0, \quad \rho > 0. \end{aligned}$$

Outcome: the joint synthesis ensures optimal DHS regulation and stable EPS–DHS coupling while bounding the weighted dynamic-port gain.

Port-Shaping Insight

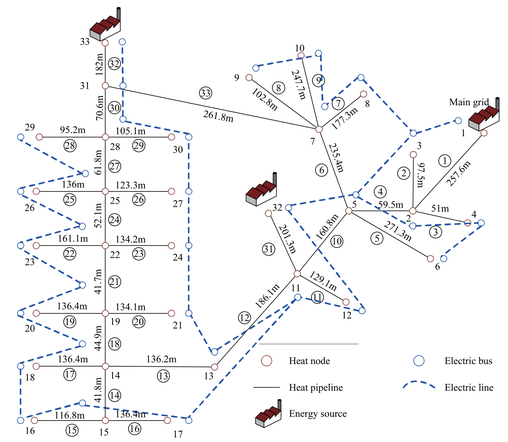
Exposing the HP-connected DHS as the grid-facing port $\omega^{HP} \mapsto p^H$ allows alternative weighting filters and additional LMI performance channels to target other transient-performance objectives.

DHS optimality
$\mathcal{L}_{\mathrm{pas}} \prec 0$
$\Downarrow$
A_{cl} Hurwitz
$\Downarrow$
$e(t) \to 0$

Stable coupling
$\mathcal{L}_{\mathrm{pas}} \prec 0$
$\Downarrow$
Stable EPS–DHS interconnection

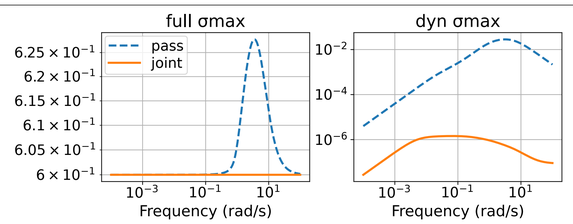
Port shaping
$\mathcal{L}_\infty \prec 0$
$\Downarrow$
Weighted dynamic attenuation

Numerical Experiments



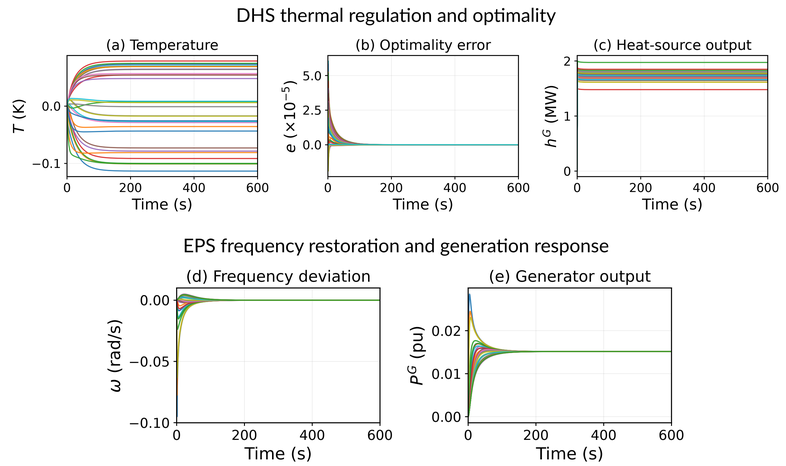
Barry Island test system with a reduced 33-bus EPS, a 33-node DHS, and three HPs. Reproduced from X. Qin et al., *CSEE J. Power Energy Syst.*, 2021.

Frequency-Domain DHS-Port Shaping



Frequency response of the DHS interface channel $\omega^{HP} \to p^H$.

Closed-Loop DHS Optimality and Frequency Restoration



Coupled EPS–DHS Frequency Support

Table 1. Joint/passivity-only performance ratio at representative disturbance frequencies.

Frequency	$J_{\mathrm{freq},L1}$	$J_{u^{EPS},L2}$	peak $\lvert h_k^G \rvert$	$J_{h^G,L2}$
0.0016 Hz	0.24	0.11	0.96	1.00
0.016 Hz	0.006	0.021	0.93	0.99
50 Hz	0.008	0.019	0.96	1.00